# Local criticality and marginal Fermi liquid in a solvable model

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*Y. Werman, D. Chowdhury, T. Senthil, and EB, arXiv:XXXX.XXXX*

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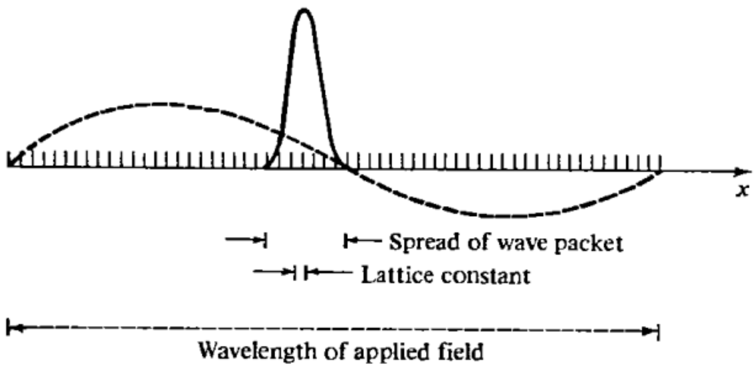
# Semiclassical theory of transport in metals

$$k_F l \gg 1$$

$$l/a \gg 1$$




Drude formula: $\rho = \dfrac{m}{ne^2\tau} = \dfrac{3\pi}{2}\dfrac{h}{e^2 k_F}\dfrac{1}{k_F l}$

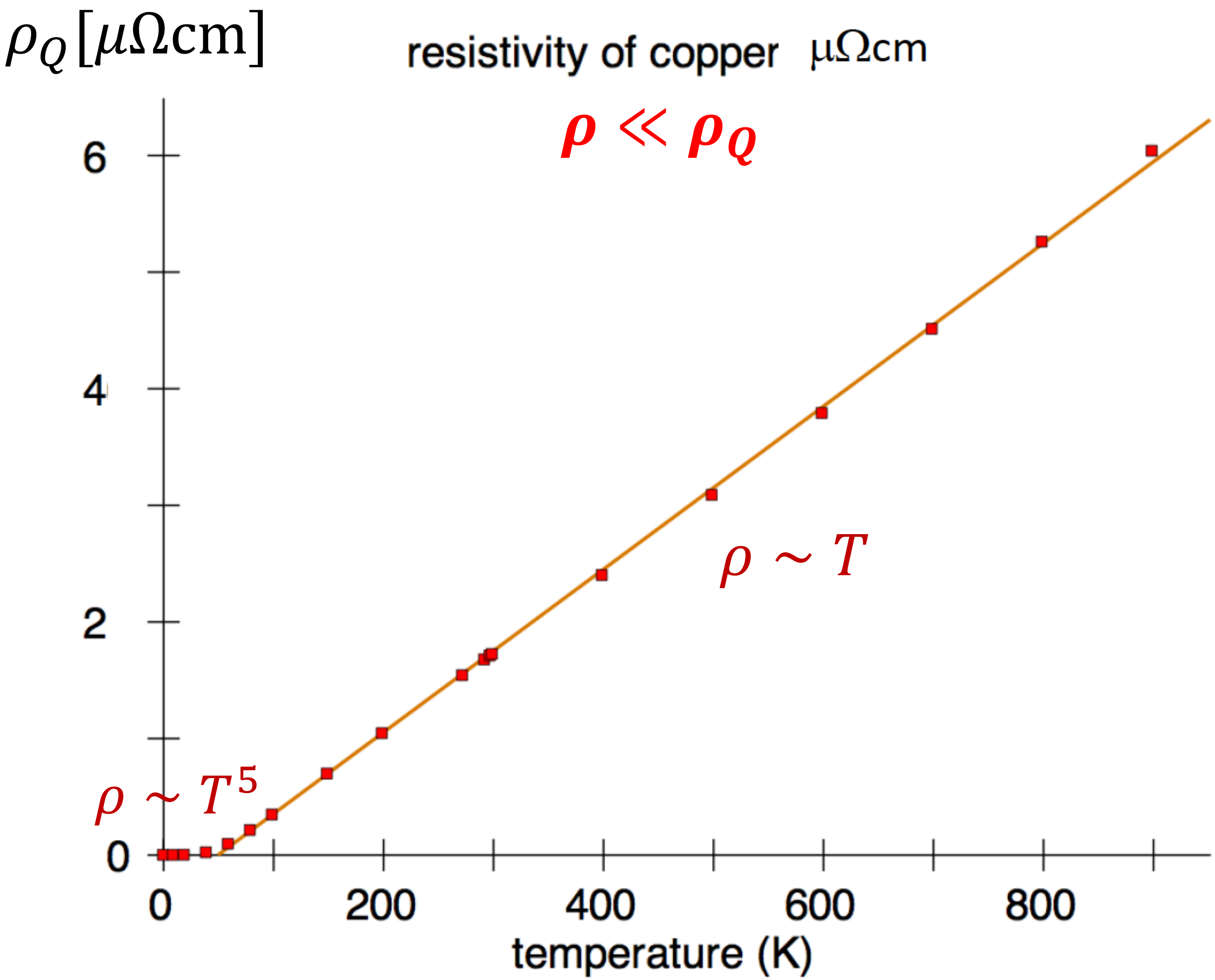
Mott-Ioffe-Regel limit

$$\rho \ll \frac{3\pi}{2}\frac{h}{e^2}\frac{1}{k_F} \equiv \left(\frac{3\pi}{2k_F a_B}\right)\rho_Q$$

**"Quantum of Resistivity"**

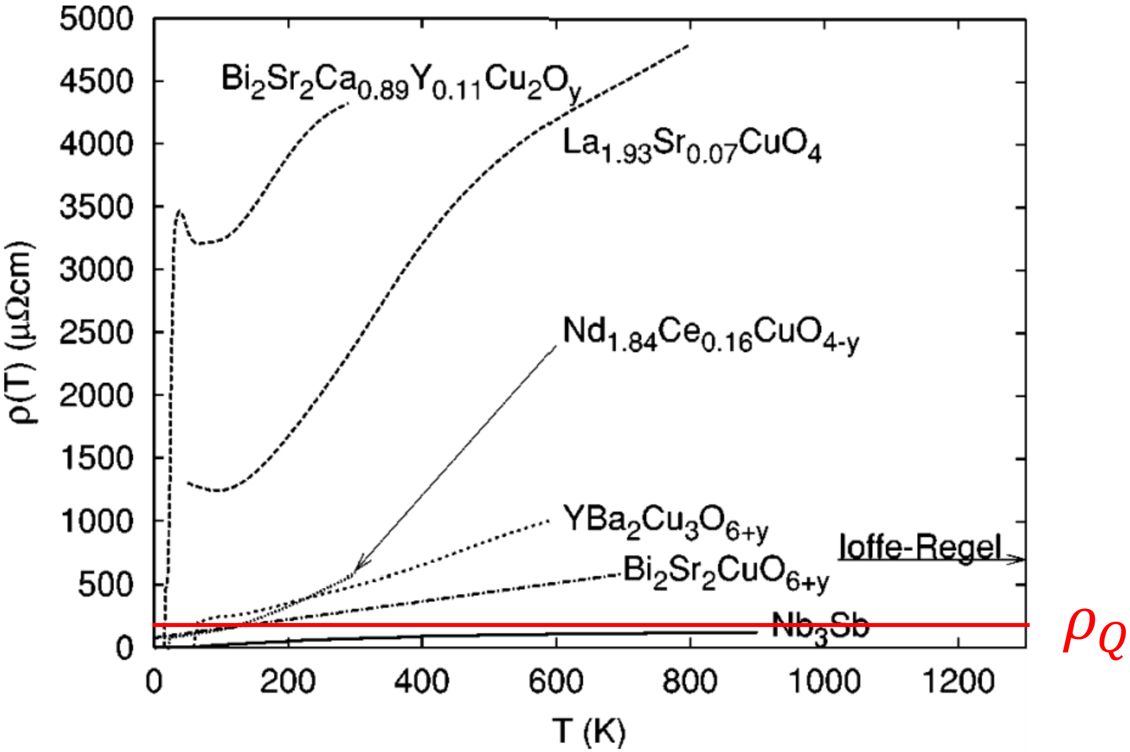
$$\rho_Q = \frac{h}{e^2}a_B = 136.6\mu\Omega\text{cm}$$

# Resistivity of a good metal



# "Bad Metals"

From Caladra and Gunnarson (2003)

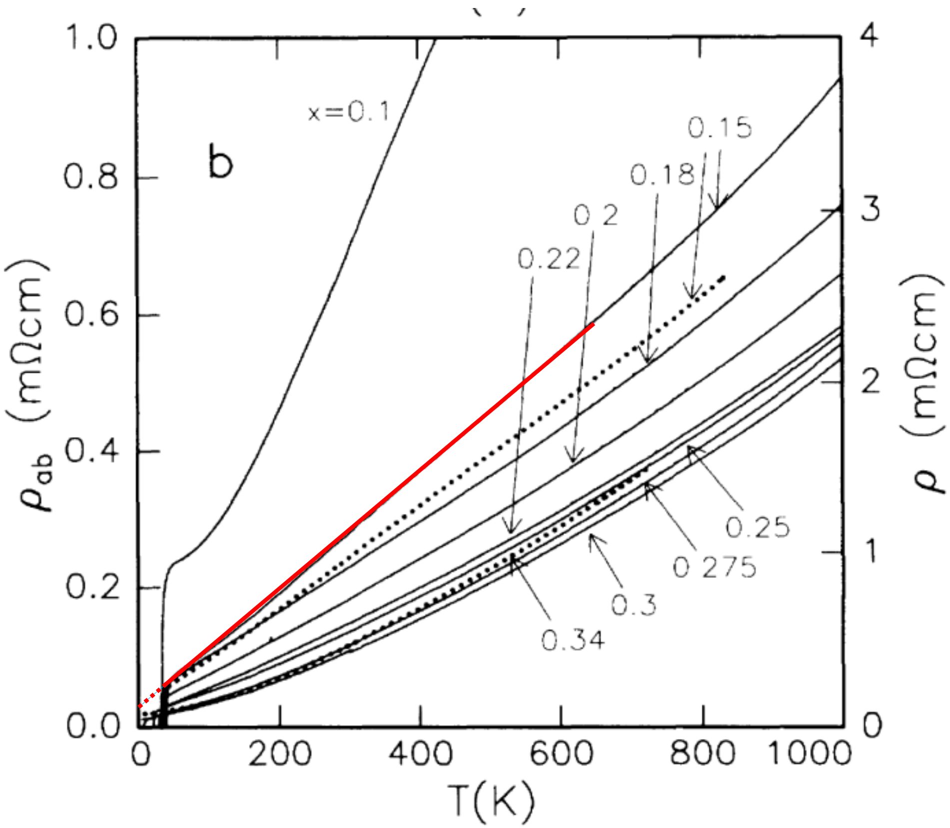



Bad metals: $\rho(T) \gtrsim \rho_Q, \frac{d\rho(T)}{dT} > 0$

**Transport without quasiparticles?**

*Emery and Kivelson, PRL (1995)*

# “Strange” metals at $T \to 0$ ?




$La_{2-x}Sr_xCuO_4$ *Takagi et al., PRL (1992)*

- “Planckian Bound” $\frac{1}{\tau} \leq \frac{\alpha k_B T}{\hbar}$ *(Sachdev, Zaanen, Hartnoll, Blake…)*
- Relation to bound on quantum chaos? *(Maldacena et al.)*
- Quantum critical point? $z = \infty$ (“local QCP”)? *(Si, Varma)*

# Theoretical challenges

- Theory for transport in “bad metal” regime, $\rho \gtrsim \rho_Q$?
- Model for $\rho \sim T$ extending down to low $T$?
- Fundamental bounds on resistivity? Role of quantum criticality?

# Outline

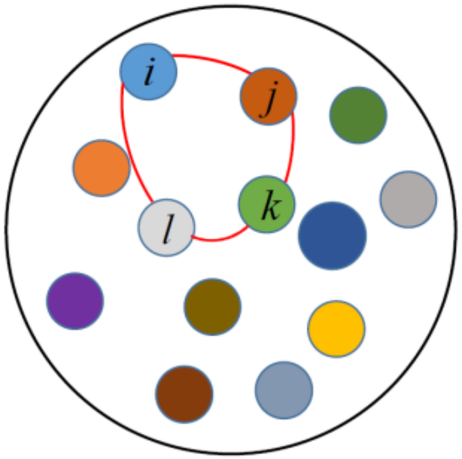
- Translationally invariant large-N model with strong e-e interactions: Fermi liquid, Marginal Fermi liquid, and non-Fermi liquid
- Implications: transport bounds, local quantum criticality

# Controlled "Non-Fermi liquid" at large N

## Sachdev-Ye-Kitaev Model:

$$H = \frac{1}{N^{3/2}} \sum_{ijkl=1}^{N} U_{ijkl} c_i^\dagger c_j^\dagger c_k c_l$$

$$\overline{U}_{ijkl} = 0, \overline{U_{ijkl}^2} = U^2$$




## Non-Fermi liquid behavior:

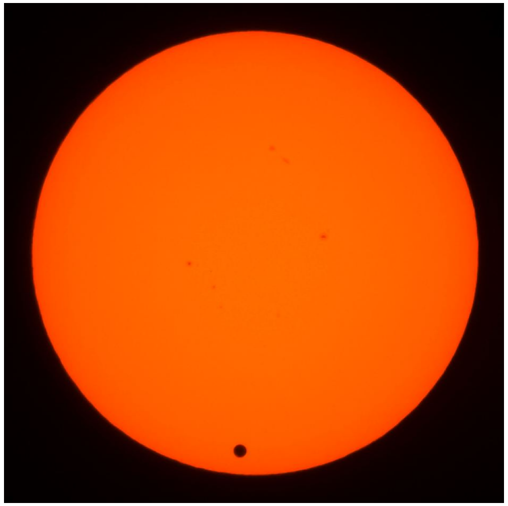
$$\overline{G_{ij}}(\omega) \sim \frac{i\mathrm{sgn}(\omega)}{\sqrt{U|\omega|}}$$

***New 'window' into non-quasiparticle transport?***

## Maximally chaotic: $\lambda_L = 2\pi T$

# Higher dimensional
# Translationally Invariant extension

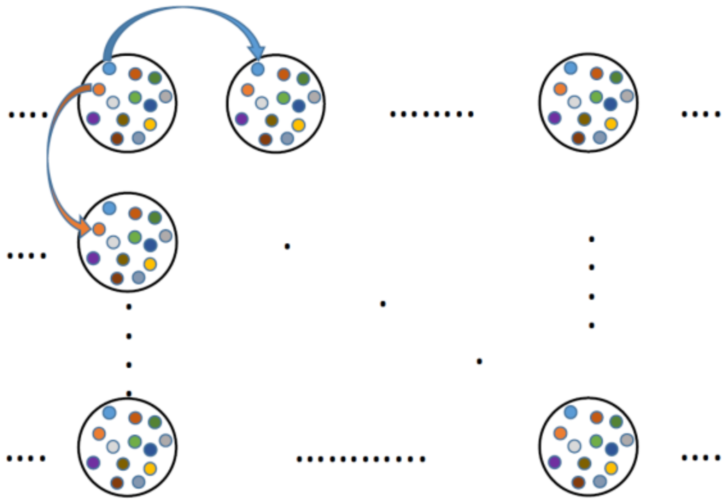
***Fermi surface***

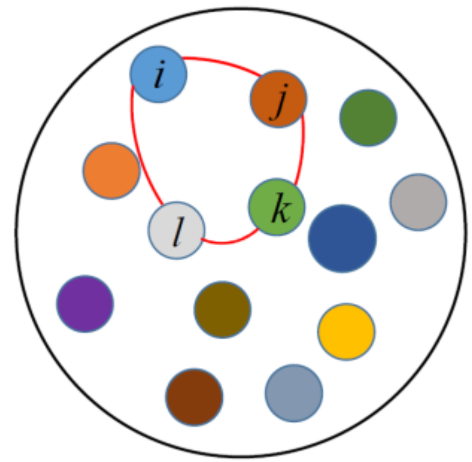


***Critical Fermi surface***

**Aug. 23, 2017**

# Higher dimensional Translationally Invariant extension






$$H = \sum_{\mathbf{k}} \sum_{i=1}^{N} \varepsilon_{\mathbf{k}} c_{\mathbf{k}i}^{\dagger} c_{\mathbf{k}i} +$$

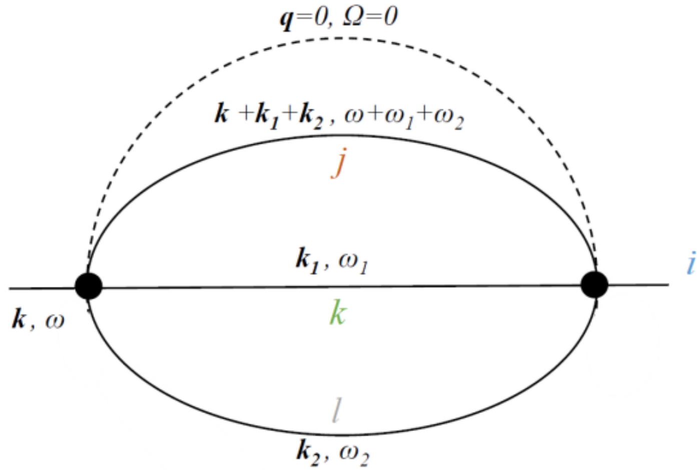
$$\frac{1}{N^{3/2}} \sum_{\mathbf{r}} \sum_{ijkl=1}^{N} U_{ijkl} c_{\mathbf{r}i}^{\dagger} c_{\mathbf{r}j}^{\dagger} c_{\mathbf{r}k} c_{\mathbf{r}l}$$

***Same $U_{ijkl}$ on every site*** $\overline{U}_{ijkl} = 0, \overline{U_{ijkl}^2} = U^2$

***Disordered lattice of SYK sites:***

***Y. Gu et al., R. Davison et al., X. Song et al. (2017)***

# One band model

$$\Sigma(\mathbf{k}, \omega) =$$ 

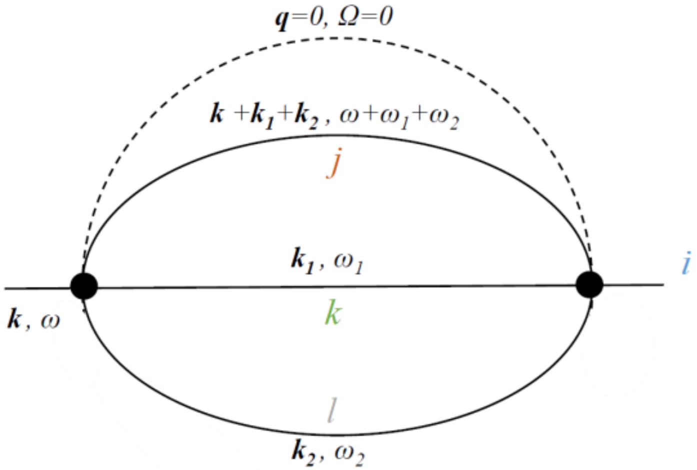

$$\Sigma(\omega_n) \sim i\frac{U}{W}\omega_n \; (\Sigma \ll W;\ \text{FL}) \qquad \omega, T \ll \frac{W^2}{U}$$

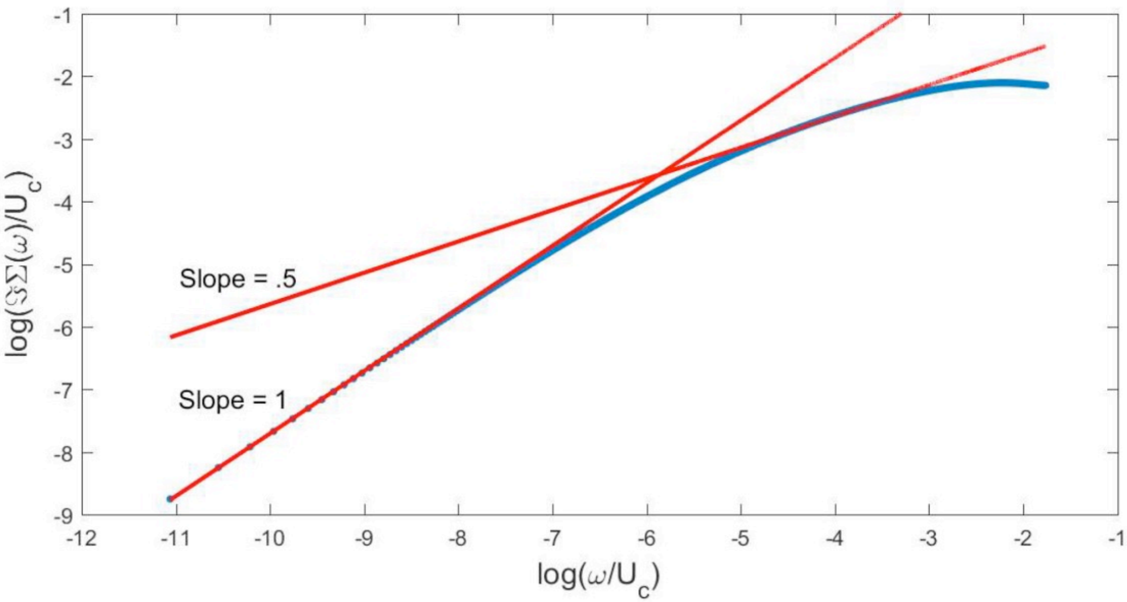
$$\Sigma(\omega_n) \sim i\sqrt{U|\omega_n|}\,\text{sgn}(\omega_n) \; (\Sigma \gg W;\ \text{FL}) \qquad \omega, T \gg \frac{W^2}{U}$$

*D. Chowdhury, et al., In Preparation (2017)*

# One band model

$$\Sigma(\mathbf{k}, \omega) =$$

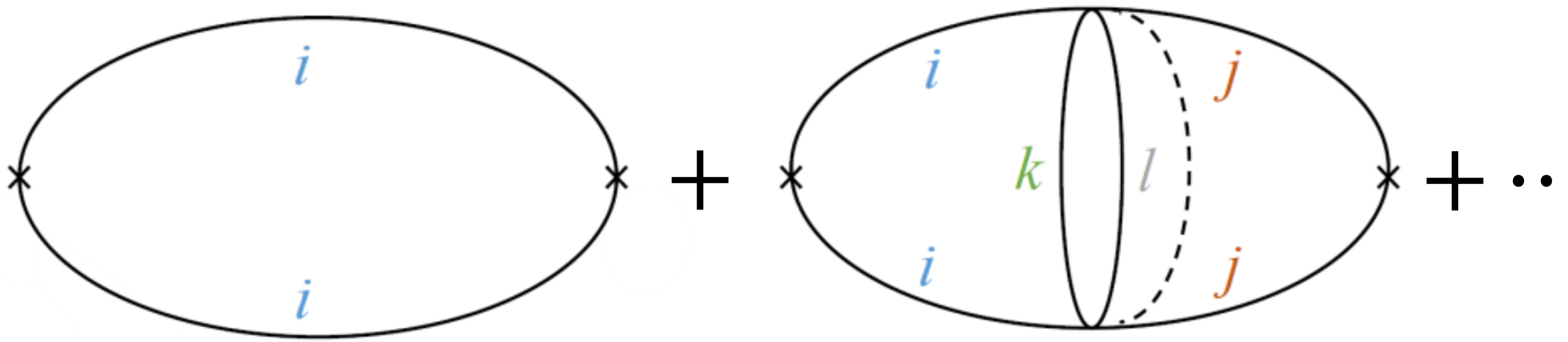


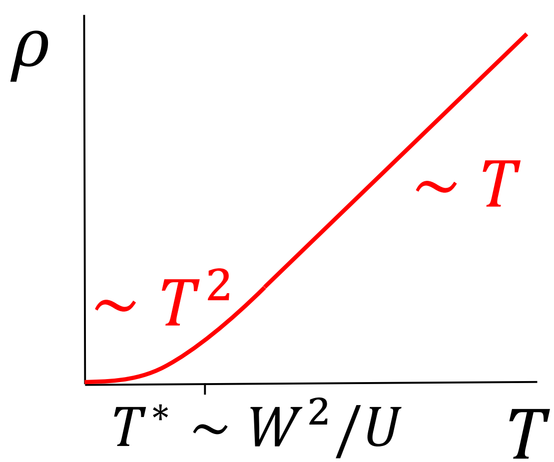




*D. Chowdhury, et al., In Preparation (2017)*

# Conductivity

$$\langle J(\omega)J(-\omega)\rangle =$$




***High T regime: No FS***

***Locally quantum critical, $\rho \sim T$***

***Critical FS***

***T=0 phase?***

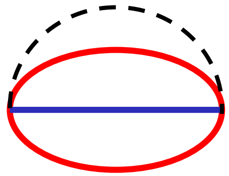
*D. Chowdhury, et al., In Preparation (2017)*

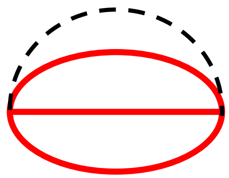
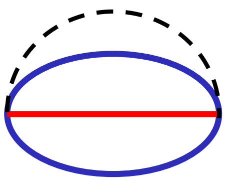
# Two band/Kondo lattice generalization

Two bands $c, f$ with bandwidths $W_f \ll W_c$

$$H = H_c + H_f + \frac{1}{N^{3/2}} \sum_{\mathbf{r}} \sum_{ijkl=1}^{N} V_{ijkl} c_{\mathbf{r}i}^{\dagger} c_{\mathbf{r}j} f_{\mathbf{r}k}^{\dagger} f_{\mathbf{r}l} +$$

$$\frac{1}{N^{3/2}} \sum_{\mathbf{r}} \sum_{ijkl=1}^{N} U_{ijkl} f_{\mathbf{r}i}^{\dagger} f_{\mathbf{r}j}^{\dagger} f_{\mathbf{r}k} f_{\mathbf{r}l}$$

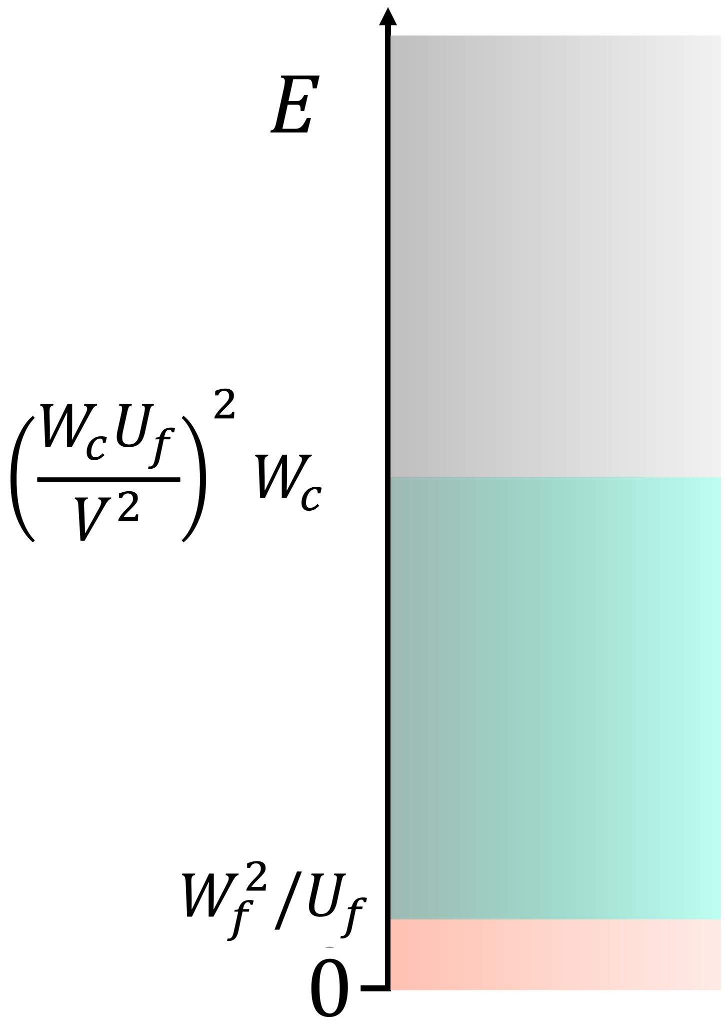
$\Sigma_c =$ 

$\Sigma_f =$  $+$ 

*D. Chowdhury, et al., In Preparation (2017)*

# Two band/Kondo lattice generalization

$U_f \gg V, W_f, W_c$




$\boldsymbol{\Sigma_f \sim \Sigma_c \sim \omega^{1/2}}$ **(SYK)**

$\boldsymbol{\Sigma_f \sim U_f^{1/2} \omega^{1/2}}$ **(SYK)**

$\boldsymbol{\Sigma_c \sim \frac{V^2}{W_c U_f} i\omega \log(\omega)}$ **(*Marginal* Fermi liquid)**

$\boldsymbol{\Sigma_f \sim \Sigma_c \sim i\omega}$ **(Fermi Liquid)**

*D. Chowdhury, et al., In Preparation (2017)*

# Marginal Fermi Liquid (MFL)

Spectral density of $f$ fluctuations:

$$\Pi''_f(\boldsymbol{q},\omega) \sim \frac{1}{U_f}\,\mathrm{atan}\left(\frac{\omega}{T}\right)$$

*Varma et al., PRL (1989); Abbamonte et al., arXiv (2017)*

$c$ fermion lifetime:

$$\frac{1}{\tau_c} \sim \Sigma''_c(\boldsymbol{k},\omega) \sim \frac{V^2}{U_f W_c}\,\max(\omega, T)$$

***No apparent***

***"Planckain" bound***

$$\boldsymbol{\rho \sim T,\ c_V \sim T\log T}$$

*D. Chowdhury, et al., In Preparation (2017)*

# Local quantum criticality at $T \rightarrow 0$?

Stability at $T \rightarrow 0$?

The local quantum critical/MFL

“phase” has $S(T \rightarrow 0) > 0$

*D. Chowdhury, et al., In Preparation (2017)*

# Local quantum criticality at $T \to 0$?

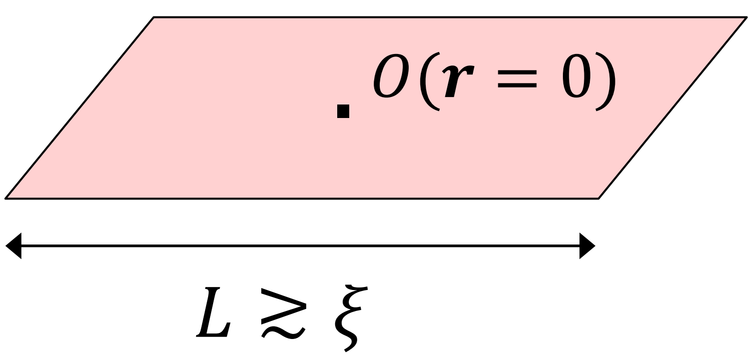
"local critical behavior" (dynamical critical exponent $z = \infty$)

is generically unstable at $T \to 0$.

$^*$*Assuming translational invariance*

E.g. assume that $\xi \sim \log(\xi_\tau)$ *Aji, Varma (2007)*

$\langle O(0,\tau)O(0,0)\rangle$




$$\xi_\tau \leq 1/\delta(L)$$

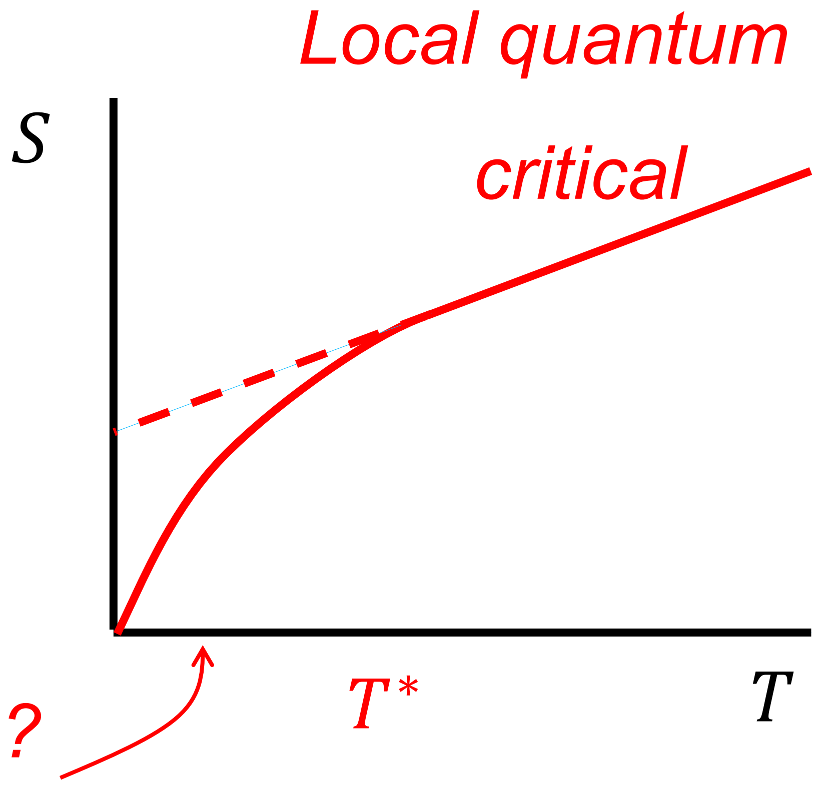
[$\delta(L)$ - Level spacing]

$$\Longrightarrow \delta(L) \leq e^{-\#L}$$

**$S(T \to 0)$ grows at least as $L$**

*D. Chowdhury, et al., In Preparation (2017)*

# Local quantum criticality at $T \rightarrow 0$?




*Fermi liquid?*

*Symmetry breaking?*

*Spin liquid phase?*

*D. Chowdhury, et al., In Preparation (2017)*

# Conclusions

**Large-N models: non-quasiparticle transport in a controlled setting.**

- Fundamental "Planckian" bound: may apply to thermalization/chaos, but not to any particular physical quantity (e.g., current)
- "Local quantum criticality": probably never a stable $T \to 0$ phase/QCP.
- "Parent" multi-critical point?
  Universality of maximally chaotic systems?

**Thank you.**